CONSTRUCTION OF PERSPECTIVE PROJECTIONS.

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The following note was suggested by an article from Professor Thornton, under the same heading, in the Annals of Mathematics, Vol. I. No. 1, in which the formulae are given for the computation of the co-ordinates of the perspective of a point referred to the centre of the picture as origin, the prime vertical and horizon as axes of the ordinate and abscissa, respectively. In the article referred to, while diagrams are given which indicate how these co-ordinates may be obtained graphically, attention is more particularly called to the arithmetical solution by use of the formulae.

It is the object of the present paper to point out the graphical solution of the same problem, in which it will be seen that only one of the perspective co-ordinates (the abscissa) will be required, the construction being effected by a method of *mixed co-ordinates.*

The system of reference being primarily the same, I quote from that article: "I refer the points of the original to an orthogonal co-ordinate system of three planes. The x-plane is the perspective plane; the y-plane is the vertical plane at right angles thereto through the centre of projection; the z-plane is the horizontal plane through the centre of projection. The distances of P from these planes are x, y, z; the perspective is referred to the axes traced by the y- and z-planes on the x-plane; and the distances of π, the perspective of P, from these axes are η, ζ. The distance of the point of view from the perspective plane is d." The radius vector of P is r, and (ρ, θ) the polar co-ordinates of π.

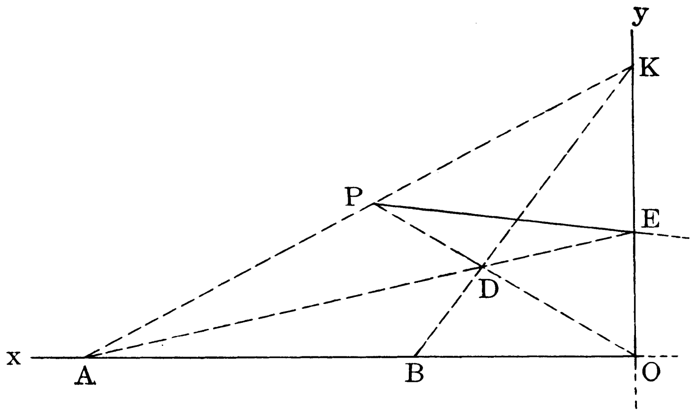
It is evident that ρ is the perspective of r, and that

$$\theta = \tan^{-1}\frac{z}{y}.$$

π is therefore located as the intersection of a line through the centre of the picture making an angle, $\tan^{-1} z/y$, with the horizon, with the vertical through $(\eta, 0)$.

In the actual construction of the perspective, no more data are needed than are required by Adhemar's Method of Scales. The plan, elevation, and perspective may be, and are best, on separate sheets of paper. On the plan, the x and y axes are located at pleasure. In elevation, the horizon is drawn in that position which produces the desired effect, or the actual elevation may be dispensed with under the same conditions as with the Method of Scales. The values of y and z are taken directly from the plan and elevation, or its equivalent, with the dividers, and the direction of ρ located in the picture by drawing *one line.* In the plan, if the point of view (C) is in reach, the line CP' (P' being the plan of any point

P) cuts off η on the y-axis, which applied to the perspective gives the position of π by *drawing* one line. If the point of view is not in reach, the following construction gives η with certainly the ease and accuracy of the vanishing point and reduced distance method, with the advantage of the auxiliary construction being on the plan and not in the picture.



In the diagram, C is on the x-axis at a distance d from O, and on the opposite side of Oy from the plan. Lay off on Ox in the direction opposite to OC the distances $OB = \dfrac{d}{1 + 2n}$, $OA = \dfrac{d}{n}$; giving to n any convenient value. The four points A, B, O, and C form an *harmonic range* and are fixed throughout the construction. If P' be any point in the plan, through P' draw AK, join KB draw OP', and through D draw AE. Then $P'E$ and the x-axis are concurrent in C, and $OE = \eta$. (*Venable's Modern Geometry*, p. 392).

If it be preferred not to locate such constructive points as K on the y-axis, then any other line through O will answer the same purpose. The value of n must be so selected as to give the positions of A and B to best suit the plan of the drawing.

In Adhemar's Method of Scales, in the perspective drawing the intersections near the horizon are oblique, here they are oblique when the points to be determined are near the prime vertical; in which case, for particular points, the values of ζ may be found instead of η by using the plan and making the same construction for the point (x, z), Oy being used as axis of z. Such points are frequently located by the intersection of the vector with some previously constructed line.

If C is within easy reach, both η and ζ may be taken from the plan and elevation as shown in Professor Thornton's diagrams, by simply drawing the two

lines which cut off these values on the axes, or otherwise by the quadrilateral construction, if C be far away, provided the elevation is such a projection as would give the desired scenic effect for the point of view fixed on the x-axis. Unfortunately for so simple a construction, elevations are usually such projections as prohibit the selection of the point of view to best advantage, whereas in the present construction, the position of the point of sight is independent of the form of the orthogonal projection in which the elevation is given, as only the *distances* of its points from the horizontal plane are required. This permits the selection of the position of C and the direction of the central ray with reference to the plan alone, and practically gives power to select any point of view we choose to take. The drawing in the perspective is as condensed as could be desired, and is practically confined to the same space for all distances of C.

SOLUTION OF EXERCISES.

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In any triangle ABC let a circle be inscribed touching the sides AB, BC, CA in N, L, M respectively. Let the centre O of this circle be joined to the vertices and from O let OP, OQ be drawn perpendicular respectively to OC, OB, and cutting BC in P and Q. Then if NP and AQ be drawn, these lines will be parallel, as will also AP and MQ. [*F. H. Loud.*]

SOLUTION.

$$\angle BOC = 90^\circ + \tfrac{1}{2}A, \text{ and } \angle C\,OP = 90^\circ;$$

$$\therefore \angle BOP = \tfrac{1}{2}A.$$

Also
$$\angle OPB = 90^\circ + \tfrac{1}{2}C = \angle AOB.$$

Hence the triangles AOB and OPB are similar, and we have

$$\overline{BO}^2 = AB \,.\, BP. \qquad (1)$$

Since $\frac{1}{2}B$ is a common angle in NBO and OBQ, these triangles are similar, and we have

$$\overline{BO}^2 = NB \,.\, BQ. \qquad (2)$$

From (1) and (2) $\dfrac{AB}{NB} = \dfrac{BQ}{BP}$; $\therefore$ NP is parallel to AQ, which was to be proved.

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